A Note on the Comparative Statics of Optimal Procurement Auctions

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Abstract

We find a sufficient condition such that a distributional upgrade on a seller's cost distribution implies a lower expected procurement cost for a buyer. We also show that even under the strongest assumption about this upgrade made in the literature so far, the seller can be worse off, even if this upgrade is costless.

1. Introduction

We consider a buyer who has to procure a service from one of n potential sellers, whose production costs are private information. We study under which circumstances it is desirable for him to face one "better" seller, in the sense that she has a better cost distribution, and when it is desirable for the seller to have such a better cost distribution. In other words, we study the comparative statics of the buyer's expected cost and seller's expected profit with respect to distributional upgrades on a seller.

For the buyer, facing a better seller is good since there is a higher probability of her having low costs but, on the other hand, it may be bad since facing a seller with a better distribution can imply that the informational rent she can extract is lower.

For the seller, having a better distribution is good since, *ceteris paribus*, it increases her probabilities of winning the auction and the informational rent she can extract. However, since this better distribution is observed by the buyer and the mechanism is changed against the better seller, there is a negative effect associated to it.

We provide a natural and weak sufficient condition on the distributional upgrade under which the buyer is better off. On the other hand, we show that for even for the strongest concept of distributional improvement used in the literature, the seller can be worse off when her cost distribution improves.

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2. Model

Consider a buyer who wants to procure a good or service and faces n potential suppliers indexed by $i = 1, ..., n$. If the buyer decides to carry out the task by himself, it would cost him an amount of money $c_0 \geq \underline{c}$. Suppliers' costs to perform the task (which are private information) are distributed independently across firms. Firm i obtains her cost from a differentiable distribution $F_i(\cdot)$, $i \geq 2$, with support $C \equiv [\underline{c}, \bar{c}]$. However, competitor 1 (from now on the *upgrader*) draws his cost from a differentiable distribution $F(\cdot, I)$ with the same support as before. I is a parameter that indexes the supplier's efficiency, and we assume that, as $I \geq 0$ increases, the distribution *improves*. For notational convenience we use $f_i(\cdot) \equiv F_i'(\cdot)$ if $i \geq 2$, and we keep using $\frac{\partial F}{\partial c}(c, I)$ in the upgrader's case.

We make the standard *regularity* assumption (first stated in [**?**]) and that guarantees the optimal mechanism can be found using pointwise maximization)

Assumption 1 *For every $i \geq 2$ and $I \geq 0$, the functions $J_i(c) = c + \frac{F_i(c)}{f_i(c)}$ and $J_I(c) = c + \frac{F(c,I)}{\frac{\partial F}{\partial c}(c,I)}$ are increasing.*

or technical reasons, we also need:

Assumption 2 *For every $c \in C$, $I \mapsto J_I^{-1}(c)$ is differentiable.*

There are several "distributional improvements" that may apply to the context presented here. We now introduce two widely-used notions, the first one being the most commonly used in statistics and economics:

Definition 3 ***(First Order Stochastic Dominance):*** *We will say that $\{F(\cdot, I)\}_{I \in \mathbb{R}_+}$ is a family of distributional improvements in the sense of first order stochastic dominance (FOSD) if, for every fixed $c \in C$, $F(c, \cdot)$ is increasing. In other words, the probability of obtaining a cost below $c \in C$ is increasing in I.*

The next one has been used before in the auction literature (see for example [[**?**]]) and was introduced first in contract theory:

Definition 4 ***(Monotone Likelihood Ratio Property):*** *We will say that $\{F(\cdot, I)\}_{I \in \mathbb{R}_+}$ is a family of distributional improvements in the sense of the monotone likelihood ratio property (MLRP) if, for every $I' < I \in \mathbb{R}_+$ and $c' < c \in C$,*

$$\frac{\frac{\partial F}{\partial c}(c', I')}{\frac{\partial F}{\partial c}(c, I')} < \frac{\frac{\partial F}{\partial c}(c', I)}{\frac{\partial F}{\partial c}(c, I)} \tag{1}$$

That is, as I increases, it is more likely to obtain lower costs relative *to higher ones. This condition is exactly to ask for $(c, I) \mapsto \frac{\partial F}{\partial c}(c, I)$ to be log-submodular.*[1]

The following well-known result relates both definitions and shows that MLRP is stronger than FOSD:

[1] A well-known result shows that MLRP implies that $\frac{F(c,I)}{\frac{\partial F}{\partial c}(c,I)}$ is increasing in I for all $c \in C$. This term corresponds to the informational rent a seller obtains when her type is c, and it increases with I, making non-trivial the comparison for the buyer: a better seller has lower costs but also extracts a higher informational rent.

Lemma 5 *If $\{F(\cdot, I)\}_{I \in \mathbb{R}_+}$ is a family of distributional improvements in the sense of MLRP, then, it is a family of distributional improvements in the sense of FOSD.*

Proof. Standard. ■

Finally, define $C^n = \{c^n = (c_1, ..., c_n) |\ c_i \in C\ \forall i = 1, ..., n\}$ and assume that for $i \geq 2,\ f_i(\cdot) > 0$ and $\forall I \geq 0,\ \frac{\partial F}{\partial c}(\cdot, I) > 0$, a.e. in C.

3. Basic Results

We now consider an upgrader with cost distribution $F(\cdot, I)$, and perform comparative statics over the procurement cost and the upgrader's utility with respect to the parameter I. The buyer's problem is to choose transfer functions $t_i : C^n \to \mathbb{R}$ (payments to the sellers) and winning probability functions $q_i : C^n \to [0, 1]$ (probabilities of buying), $i = 1, ..., n$. Under the regularity assumptions, it is direct that the expected optimal mechanism corresponds to (see [**?**])

$$q_1^*(c_1, ..., c_n) = \begin{cases} 1 & J_I(c_1) \leq \min\{c_0, J_i(c_i) |\ i \geq 2\} \\ 0 & \sim \end{cases} \tag{2}$$

$$q_i^*(c_1, ..., c_n) = \begin{cases} 1 & J_i(c_i) < \min\{c_0, J_I(c_1), J_l(c_l) |\ l \neq i,\ l \geq 2\} \\ 0 & \sim \end{cases} \tag{3}$$

$i = 2, ..., n$.

which yields a procurement cost of:

$$\mathcal{C}(I) \quad = \quad \int\limits_{C^n} \left[J_I(c_1) q_1^*(c^n) + \sum_{l \geq 2} J_l(c_l) q_l^*(c^n) + c_0 \left(1 - \sum_{i \geq 1} q_i(c^n) \right) \right] \frac{\partial F}{\partial c_1}(c_1, I) \left(\prod_{j \geq 2} f_j(c_j) \right) dc^n \tag{4}$$

Our main purpose is to establish if conditions FOSD or MLRP on the family $\{F(\cdot, I)\}_{I \geq 0}$ imply that the expected procurement cost reduces. The main proposition, stated below, shows that even FOSD implies the result.

Proposition 6 *Suppose that for every $c \in C$ the function $F(c, \cdot)$ is differentiable. A sufficient pointwise condition on the the family $\{F(\cdot, I)\}_{I \geq 0}$ under which the expected procurement cost reduce is:*

$$\forall I \geq 0,\ \forall c \in [\underline{c}, J_I^{-1}(c_0)],\ \frac{\partial F}{\partial I}(c, I) \geq 0 \tag{5}$$

As a consequence, if the mentioned family satisfies FOSD, the expected procurement cost decreases when facing a better competitor.

Proof. See Appendix. ■

As we can see, the trade-off mentioned in the introduction (a buyer likes better sellers since they have in average lower costs, but on the other hand they can extract higher informational rents) always works in the buyer's favor. This is true since he modifies the mechanism in such a way that takes advantage optimally of this distributional improvement.

4. Example

In this section we show that it is not true that the upgrader is better off when improving his distribution. Since under some specific upgrades the buyer may also extract more rent from the seller, which may out-weight the benefits related to a lower expected cost, it is possible that the seller is worse off, even if this distributional upgrade is for free.

Suppose $n = 2$, $C = [0, 1]$ and $c_0 = +\infty$. Consider $F_2(c) = c$ and $F(c, I) = c^{\frac{1}{1+I}}$, $I \geq 0$. This last family of distributions satisfies MLRP and, as a consequence, FOSD. The upgrader's expected utility when his distribution is $F(\cdot, I)$ corresponds to

$$\Pi(I) = \int\limits_C \Pi(c, c)\frac{\partial F}{\partial c}(c, I)dc = \int\limits_C Q^*(c)F(c, I)dc$$

with $Q^*(c) = \int\limits_C q^*(c, s)f(s)ds$. Using that $q^*(c, s) = 1 \Leftrightarrow J_I(c) \leq J_2(s)$ (from the previous characterization of the optimal mechanism), $J_I(c) = c(2 + I)$ and $J_2(c) = 2c$, (thus $J_2^{-1}(J_I(c)) = \frac{c(2+I)}{2}$) we have

$$\begin{aligned}
\Pi(I) &= \int\limits_{\underline{c}}^{\frac{2}{2+I}} \left[1 - \frac{c(2+I)}{2}\right] c^{\frac{1}{2+I}} dc \\
&= \frac{(1+I)^2}{(2+I)(3+2I)} \left(\frac{2}{2+I}\right)^{\frac{2+I}{1+I}}
\end{aligned}$$

The next result establishes that no positive investment level is profitable for the seller. Even though investment increase profits by reducing the expected cost in the case of winning the competition, this effect is out-weighted by more disadvantageous rules imposed by the buyer, which permit him to extract a larger fraction of the seller's informational rent.

Proposition 7 *For all* $I \geq 0$, $\frac{d}{dI}(\Pi(I)) < 0$.

Proof. See Appendix ■

5. Appendix: Proofs

We first rewrite the procurement cost in the next lemma:

Lemma 8 *The expected procurement cost can be written as*

$$\begin{aligned}\mathcal{C}(I) &= \sum_{i\geq 2}\int_{\underline{c}}^{J_i^{-1}(c_0)} f_i(c)\left(\prod_{l\neq i}[1-F_l(J_l^{-1}(J_i(c)))]\right)J_I^{-1}(J_i(c))F(J_I^{-1}(J_i(c)),I)dc \\ &+\left(\prod_{l\geq 2}[1-F_l(J_l^{-1}(c_0))]\right)J_I^{-1}(c_0)F(J_I^{-1}(c_0),I) \\ &+\sum_{i\geq 2}\int_{\underline{c}}^{J_i^{-1}(c_0)} f_i(c)\left(\prod_{l\neq i}[1-F_l(J_l^{-1}(J_i(c)))]\right)J_i(c)[1-F(J_I^{-1}(J_i(c)),I)]dc \\ &+c_0\left(\prod_{l\geq 2}[1-F_l(J_l^{-1}(c_0))]\right)[1-F(J_I^{-1}(c_0),I)] \qquad (6)\end{aligned}$$

Proof Lemma ??: Define $H(c^n,I)$ as

$$H(c^n,I)\equiv\left[J_I(c_1)q_1^*(c^n)+\sum_{l\geq 2}J_l(c_l)q_l^*(c^n)+c_0\left(1-\sum_{i\geq 1}q_i^*(c^n)\right)\right]\frac{\partial F}{\partial c_1}(c_1,I)\prod_{i\geq 2}f_i(c_i)$$

and consider the set

$$A=\{c^n\in C^n|\ J_I(c_1)\leq c_0,\ J_I(c_1)\leq J_i(c_i),\ \forall i\geq 2\}$$

That is, it is the set of cost-vectors in which the *upgrader* wins the procurement auction. Therefore,

$$\mathcal{C}(I)=\int_A H(c^n,I)dc^n+\int_{C^n\setminus A}H(c^n,I)dc^n$$

Set A can be written as $A=A_0\cup\left(\bigcup_{i\geq 2}A_i\right)$ with

$$\begin{aligned}A_0 &= \{c^n\in C^n|\ J_I(c_1)\leq c_0\ \wedge\ c_0<J_i(c_i),\ \forall i\geq 2\} \\ &= \{c^n\in C^n|\ c_1<J_I^{-1}(c_0)\ \wedge\ J_i^{-1}(c_0)\leq c_i,\ \forall i\geq 2\}\end{aligned}$$

$$\begin{aligned}A_i &= \{c^n\in C^n|\ J_I(c_1)\leq J_i(c_i)\ \wedge\ J_i(c_i)\leq c_0\ \wedge\ (J_i(c_i)\leq J_l(c_l),\ l\geq i)\ \wedge\ (J_i(c_i)<J_l(c_l),\ i>l)\} \\ &= \{c^n\in C^n|\ c_1\leq J_I^{-1}(J_i(c_i))\ \wedge c_i\leq J_i^{-1}(c_0)\ \wedge\ (J_l^{-1}(J_i(c_i))\leq c_l,\ l\geq i)\ \wedge\ (J_l^{-1}(J_i(c_i))<c_l,\ i>l)\}\end{aligned}$$

and it is quite easy to see that $A_j\cap A_i=\emptyset$ if $i\neq j$ $i,j\in\{0,2,3,...,n\}$. Note that in A_i the *upgrader* wins the procurement auction and seller i reports de lowest virtual cost among all the upgrader's rivals.

On the other hand, in A_0 the same agent wins the competition but no other firm submits a bid below the reserve cost c_0. Implicitly in our above definitions, among the lowest virtual costs, the upgrader wins the procurement auction, which certainly doesn't increase expected expenditures for the buyer. As a direct consequence,

$$\int_A H(c^n, I) dc^n = \sum_{i=0, i\geq 2} \int_{A_i} H(c^n, I) dc^n$$

Now, define $t_l(\cdot) \equiv J_l^{-1}(J_i(\cdot))$ for $l \geq 2,\ l \neq i$ and $t_I(\cdot) \equiv J_I^{-1}(J_i(\cdot))$ Integrating over A_i yields

$$\int_{A_i} H(c^n, I) dc^n = \int_{\underline{c}}^{J_i^{-1}(c_0)} \int_{t_2(c_i)}^{\bar{c}} \ldots \int_{t_{i-1}(c_i)}^{\bar{c}} \int_{t_{i+1}(c_i)}^{\bar{c}} \ldots \int_{t_n(c_i)}^{\bar{c}} \int_{\underline{c}}^{t_I(c_i)} J_I(c_1) \frac{\partial F}{\partial c_1}(c_1, I) \left(\prod_{l \geq 2} f_l(c_l) \right) dc^n$$

and observing that $J_I(c_1) \frac{\partial F}{\partial c_1}(c_1, I) = \left[c_1 + \frac{F(c_1, I)}{\frac{\partial F}{\partial c_1}(c_1, I)} \right] \frac{\partial F}{\partial c_1}(c_1, I) = \frac{d}{dc_1}(c_1 F(c_1, I))$ we obtain

$$\int_{A_i} H(c^n, I) dc^n = \int_{\underline{c}}^{J_i^{-1}(c_0)} f_i(c) \left(\prod_{l \neq i} [1 - F_l(J_l^{-1}(J_i(c)))] \right) J_I^{-1}(J_i(c)) F(J_I^{-1}(J_i(c)), I) dc$$

Analogously,

$$\begin{aligned} \int_{A_0} H(c^n, I) dc^n &= \int_{J_2^{-1}(c_0)}^{\bar{c}} \ldots \int_{J_n^{-1}(c_0)}^{\bar{c}} \int_{\underline{c}}^{J_I^{-1}(c_0)} J_I(c_1) \frac{\partial F}{\partial c_1}(c_1, I) \left(\prod_{l \geq 2} f_l(c_l) \right) dc^n \\ &= \left(\prod_{l \geq 2} [1 - F_l(J_l^{-1}(c_0))] \right) J_I^{-1}(c_0) F(J_I^{-1}(c_0), I) \qquad (7) \end{aligned}$$

Thus,

$$\begin{aligned} \int_A H(c^n, I) dc^n &= \sum_{i \geq 2} \int_{\underline{c}}^{J_i^{-1}(c_0)} f_i(c) \left(\prod_{l \neq i} [1 - F_l(J_l^{-1}(J_i(c)))] \right) J_I^{-1}(J_i(c)) F(J_I^{-1}(J_i(c)), I) dc \\ &+ \left(\prod_{l \geq 2} [1 - F_l(J_l^{-1}(c_0))] \right) J_I^{-1}(c_0) F(J_I^{-1}(c_0), I) \qquad (8) \end{aligned}$$

On the other hand,

$$C^n \setminus A^n = \{c^n \in C^n |\ (\exists j \geq 2,\ J_l(c_l) < J_I(c_1)\ \wedge\ J_l(c_l) \leq c_0)\ \vee (c_0 < J_I(c_1),\ c_0 < J_i(c_i),\ \forall i \geq 2)\}$$

is the set over which the upgrader loses the procurement auction. As before, this set can be partitioned as $C^n \setminus A = B_0 \cup \left(\bigcup_{j \geq 2} B_j \right)$ with

$$B_0 = \{c^n \in C^n |\ J_I^{-1}(c_0) < c_1\ \wedge\ J_i^{-1}(c_0) < c_i,\ \forall i \geq 2\} \qquad (9)$$

$$B_i = \{c^n \in C^n |\ c_i \leq J_i^{-1}(c_0)\ \wedge\ (J_j^{-1}(J_i(c_i)) \leq c_l,\ i \leq l)\ \wedge (J_l^{-1}(J_i(c_i)) < c_l,\ i < l)\ \wedge\ J_I^{-1}(J_i(c_i)) < c_1\}$$

Set B_0 represents the zone in which the project is not assigned and B_i corresponds to the region where firm $i \geq 2$ wins the competition. Implicitly in the definition of these sets we assume that, in case of equal lowest-virtual-costs, the task is assigned to the lowest-index competitor, which certainly doesn't increase expected procurement expenditures. Then we can write

$$\int_{C^n \setminus A} H(c^n, I) dc^n = \sum_{i=0, i\geq 2} \int_{B_i} H(c^n, I) dc^n$$

It is direct that

$$\begin{aligned}
\int_{B_i} H(c^n, I) dc^n &= \int_{\underline{c}}^{J_i^{-1}(c_0)} \int_{t_2(c_i)}^{\bar{c}} \dots \int_{t_{i-1}(c_i)}^{\bar{c}} \int_{t_{i+1}(c_i)}^{\bar{c}} \dots \int_{t_n(c_i)}^{\bar{c}} \int_{t_I(c_i)}^{\bar{c}} J_i(c_i) \frac{\partial F}{\partial c_1}(c_1, I) \left(\prod_{l \geq 2} f(c_l) \right) dc^n \\
&= \int_{\underline{c}}^{J_i^{-1}(c_0)} \left(\prod_{l \neq i} [1 - F_l(J_l^{-1}(J_i(c_i)))] \right) [c_i f_i(c_i) + F_i(c_i)][1 - F(J_I^{-1}(J_i(c_i)), I)] dc_i \\
&= \int_{\underline{c}}^{J_i^{-1}(c_0)} f_i(c) \left(\prod_{l \neq i} [1 - F_l(J_l^{-1}(J_i(c)))] \right) J_i(c)[1 - F(J_I^{-1}(J_i(c)), I)] dc \qquad (10)
\end{aligned}$$

Also,

$$\begin{aligned}
\int_{B_i} H(c^n, I) dc^n &= \int_{J_2^{-1}(c_0)}^{\bar{c}} \dots \int_{J_n^{-1}(c_0)}^{\bar{c}} \int_{J_I^{-1}(c_0)}^{\bar{c}} c_0 \frac{\partial F}{\partial c_1}(c_1, I) \left(\prod_{l \geq 2} f_l(c_l) \right) dc^n \\
&= c_0 \left(\prod_{l \geq 2} [1 - F_l(J_l^{-1}(c_0))] \right) [1 - F(J_I^{-1}(c_0), I)] \qquad (11)
\end{aligned}$$

As a consequence,

$$\begin{aligned}
\int_{C^n \setminus A} H(c^n, I) dc^n &= \sum_{i \geq 2} \int_{\underline{c}}^{J_i^{-1}(c_0)} f_i(c) \left(\prod_{l \neq i} [1 - F_l(J_l^{-1}(J_i(c)))] \right) J_i(c)[1 - F(J_I^{-1}(J_i(c)), I)] dc \\
&\quad + c_0 \left(\prod_{l \geq 2} [1 - F_l(J_l^{-1}(c_0))] \right) [1 - F(J_I^{-1}(c_0), I)] \qquad (12)
\end{aligned}$$

which concludes the proof.

□

Proof of Proposition ??: Define

$$\alpha_i(c) \equiv f_i(c) \left(\prod_{l \neq i} [1 - F_l(J_j^{-1}(J_i(c)))] \right)$$

thus,

$$\mathcal{C}(I) = \sum_{i\geq 2} \int_{\underline{c}}^{J_i^{-1}(c_0)} \alpha_i(c)\{J_I^{-1}(J_i(c))F(J_I^{-1}(J_i(c)),I) + [1-F(J_I^{-1}(J_i(c)),I)]J_i(c)\}dc \tag{13}$$

$$+ \left(\prod_{l\geq 2}[1-F_l(J_l^{-1}(c_0))]\right)\{J_I^{-1}(c_0)F(J_I^{-1}(c_0),I) + [1-F(J_I^{-1}(c_0),I)]c_0\} \tag{14}$$

Therefore, under suitable integrability conditions

$$\mathcal{C}'(I) = \sum_{i\geq 2} \int_{\underline{c}}^{J_i^{-1}(c_0)} \alpha_i(c)\frac{\partial}{\partial I}\{F(J_I^{-1}(J_i(c)),I)[J_I^{-1}(J_i(c)) - J_i(c)]\}dc \tag{15}$$

$$+ \left(\prod_{l\geq 2}[1-F_l(J_l^{-1}(c_0))]\right)\frac{\partial}{\partial I}\{F(J_I^{-1}(c_0),I)[J_I^{-1}(c_0) - c_0]\} \tag{16}$$

Define $L(c,I) \equiv F(J_I^{-1}(c),I)[J_I^{-1}(c) - c]$. Thus,

$$\begin{aligned}\frac{\partial L}{\partial I}(c,I) &= \left[\frac{\partial F}{\partial t}(J_I^{-1}(c),I)\frac{\partial}{\partial I}(J_I^{-1}(c)) + \frac{\partial F}{\partial I}(J_I^{-1}(c),I)\right][J_I^{-1}(c) - c] \\ &\quad + F(J_I^{-1}(c),I)\frac{\partial}{\partial I}(J_I^{-1}(c)) \\ &= \frac{\partial}{\partial I}(J_I^{-1}(c))\left[\frac{\partial F}{\partial t}(J_I^{-1}(c),I)[J_I^{-1}(c) - c] + F(J_I^{-1}(c),I)\right] \\ &\quad + \frac{\partial F}{\partial I}(J_I^{-1}(c),I)[J_I^{-1}(c) - c] \end{aligned} \tag{17}$$

Recall that $J_I(t) = t + \frac{F(t,I)}{\frac{\partial F}{\partial t}(t,I)}$, so, evaluating at $t = J_I^{-1}(c)$ we obtain

$$J_I^{-1}(c) - c = -\frac{F(J_I^{-1}(c),I)}{\frac{\partial F}{\partial t}(J_I^{-1}(c),I)}$$

Thus,

$$\frac{\partial L}{\partial I}(c,I) = \frac{\partial F}{\partial I}(J_I^{-1}(c),I)[J_I^{-1}(c) - c] \tag{18}$$

Therefore,

$$\mathcal{C}'(I) = \sum_{i\geq 2} \int_{\underline{c}}^{J_i^{-1}(c_0)} \alpha_i(c)\frac{\partial F}{\partial I}(J_I^{-1}(J_i(c)),I)[J_I^{-1}(J_i(c)) - J_i(c)]dc \tag{19}$$

$$+ \left(\prod_{l\geq 2}[1-F_l(J_l^{-1}(c_0))]\right)\frac{\partial F}{\partial I}(J_I^{-1}(c_0),I)[J_I^{-1}(c_0) - c_0] \tag{20}$$

Since $\alpha_i(c) \geq 0$ and $J_I^{-1}(c) - c \leq 0$, $\forall c \in C$, a sufficient condition to obtain $\mathcal{C}'(I) \leq 0$ is

$$\forall i \geq 2,\ \forall c \in [\underline{c}, J_i^{-1}(c_0)],\ \frac{\partial F}{\partial I}(J_I^{-1}(J_i(c)), I) \geq 0$$

and

$$\frac{\partial F}{\partial I}(J_I^{-1}(c_0), I) \geq 0$$

which are equivalent to

$$\forall c \in [\underline{c}, J_I^{-1}(c_0)],\ \frac{\partial F}{\partial I}(c, I) \geq 0$$

since $J_I(\underline{c}) = J_i(\underline{c}) = \underline{c}$ and $J_I(\cdot)$ and $J_i(\cdot)$, $i \geq 2$, are increasing functions.

□

Proof of Proposition **??**: In order to show that $\Pi(I)$ is strictly decreasing for all $I \geq 0$, we will prove that $\frac{d}{dI}(log(\Pi(I))) < 0$ which is obviously equivalent. Recall that

$$\begin{aligned}\Pi(I) &= \int_{\underline{c}}^{\frac{2}{2+I}} \left[1 - \frac{c(2+I)}{2}\right] c^{\frac{1}{2+I}} dc \\ &= \frac{(1+I)^2}{(2+I)(3+2I)} \left(\frac{2}{2+I}\right)^{\frac{2+I}{1+I}}\end{aligned}$$

so, we have that $log(\Pi(I))$ satisfies

$$\frac{d}{dI}(log(\Pi(I))) = \frac{1}{1+I} - \frac{1}{2+I} - \frac{2}{3+2I} + \frac{1}{(1+I)^2}\left[log\left(\frac{2+I}{2}\right)\right]$$

Also, since $log(x) \leq x - 1$ (with equality only at $x = 1$), it is direct that $log\left(\frac{2+I}{2}\right) \leq \frac{I}{2}$. Then,

$$\begin{aligned}\frac{d}{dI}(log(\Pi(I))) &\leq \frac{1}{1+I} - \frac{1}{2+I} - \frac{2}{3+2I} + \frac{1}{(1+I)^2}\frac{I}{2} \\ &= \frac{2(1+I)(2+I)(3+2I) - 2(3+2I)(1+I)^2 - 4(2+I)(1+I)^2 + I(2+I)(3+2I)}{2(2+I)(3+2I)(1+I)^2} \\ &\equiv \frac{g(I)}{h(I)}\end{aligned}$$

As a consequence it suffices to show that $g(I)$ is negative for any possible investment level, since $h(I) > 0$ always. After straightforward algebra we obtain that $g(I) = -2I^3 - 5I^2 - 4I - 2$ which is certainly strictly less that zero, concluding the proof.

□

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