Sequential Procurement Auctions and Their Effect on Investment Decisions

Gonzalo Cisternas
MIT Sloan

Nicolas Figueroa
PUC-Chile
Procurement Auctions

- Markets designed for the purchase of goods (typically of high cost)
- Used both in public and private sector
- Finding ways to reduce total expenditures is a question of first-order relevance:
  - OECD countries’ public procurement expenditures in 2011 accounted for 19% of their GDP
  - Chile: Transactions performed through Chilecompra 10.000 million USD in 2013 (≈ 4% GDP)
  - Also a relevant question in the private sector
Main Features

- These mechanisms are used repeatedly over time
- Tasks sometimes involve a high degree of expertise (know-how) ⇒ Group of sellers does not change too much
- Sellers can invest in improving their technologies. Specialized tasks ⇒ Relationship-specific investments
Two ways through which total expenditures can be reduced are:

1. Inter-temporal incentives: design of **dynamic mechanisms** that smooth out costs across time
2. Incentivizing sellers to **invest in cost-reducing technologies**

We derive the **optimal contract** (i.e. optimal auction + optimal level of investment) chosen by a buyer in an environment where:

- She must purchase two goods sequentially over time and can fully commit to a two-period mechanism
- The winner of the first auction can invest in a cost-reducing technology for the second auction
Main Results

- The optimal mechanism gives an advantage to the first-period winner in the second auction.
  - Advantage decreases with the number of sellers, but it never disappears.
- In this dynamic setting, commitment induces over-investment.
- Investment observability is irrelevant for cost minimization and surplus maximization.
- More generally, in dynamic environments awarding advantages:
  - Can induce more competition among sellers $\Rightarrow$ reduce current costs.
  - Can incentivize sellers to invest more in cost-reducing technologies $\Rightarrow$ reduce future costs.


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- Efficiency
- Cost Minimization
- Conclusions
- Lack of Commitment
Basics

- A buyer (she) must purchase two goods sequentially over time.
- There are $n$ risk-neutral sellers that are ex-ante identical.
- A Seller’s cost to produce each good is his private information.
- Costs are independent across sellers, and also independent across time.
- We are interested in mechanism design, i.e., the buyer can commit to a two-period mechanism at time zero.
  - Since costs are i.i.d. across time, the revelation principle also holds when the buyer lacks commitment.
In the first period a seller’s cost is drawn from a c.d.f. $F(\cdot)$, with density $f(\cdot)$ and support $C = [c, \bar{c}]$.

First-period losers maintain $F(\cdot)$ for the second period.

The first-period winner instead can invest in a cost-reducing technology between auctions:

- Investing $I \geq 0 \Rightarrow$ Cost distribution becomes $G(\cdot, I)$, with density $g(\cdot, I)$ and support $C'$.
- Investing is costly: $\Psi : \mathbb{R}_+ \to \mathbb{R}_+$ differentiable, strictly increasing and strictly convex, with $\Psi(0) = \Psi'(0) = 0$. 
Regularity Assumptions Over $F(\cdot)$ and $G(\cdot, \cdot)$

Assumption:

(i) $c + F(c)/f(c)$ is strictly increasing in $c$.

(ii) $F(c) \leq G(c, 0)$ for all $c \in C$.

(iii) For each $c \in C$, $I \mapsto G(c, I)$ is twice continuously differentiable, strictly increasing (FOSD) and concave. Furthermore, $\frac{\partial G}{\partial I}(c, 0) > 0$ for all $c \in C$.

**Obs:** The following are sufficient for (ii) and (iii):

(a) MLRP: For all $c' < c \in C$ and $0 \leq I' < I \in \mathbb{R}$,

$$\frac{f(c')}{f(c)} \leq \frac{g(c', I')}{g(c, I')} < \frac{g(c', I)}{g(c, I)}.$$

(b) Hazard-rate ordering: For all $c \in C$ and $0 \leq I' < I$

$$\frac{g(c, I)}{G(c, I)} \leq \frac{g(c, I')}{G(c, I')} \leq \frac{f(c)}{F(c)}.$$
Timeline

- \( t=0 \): The rules of both procurement auctions are set

- \( t=1 \): First procurement auction takes place

- \( t=2 \): (1) Investment takes place. (2) Second procurement auction takes place
Direct Mechanisms

Definition

A direct mechanism that implements \( I \geq 0, \Gamma(I) \), corresponds to a tuple \( \Gamma(I) = (t^1(\cdot), q^1(\cdot), t^2_w(\cdot; I), q^2_w(\cdot; I), t^2_ℓ(\cdot; I), q^2_ℓ(\cdot; I)) \) where

\[
\begin{align*}
t^1 & : C^n \rightarrow \mathbb{R}^n \quad \text{(transfer at } t=1) \\
q^1 & : C^n \rightarrow \Delta_n \quad \text{(allocation rule at } t=1) \\
t^2_w(\cdot; I) & : C^n \rightarrow \mathbb{R} \\
q^2_w(\cdot; I) & : C^n \rightarrow [0, 1] \\
t^2_ℓ(\cdot; I) & : C^n \rightarrow \mathbb{R}^{n-1} \\
q^2_ℓ(\cdot; I) & : C^n \rightarrow [0, 1]^{n-1}
\end{align*}
\]

such that \( q^2_w(c; I) + \sum_{i \neq w} q^2_ℓ,i(c; I) = 1 \) for all \( c \in C^n \), and such that the first-period winner finds it optimal to invest \( I \geq 0 \) between auctions.
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Ex-Post Allocative Efficiency

- Planner observes \( I \) and realized costs, and maximizes **total surplus**
- Efficient mechanism \( \Gamma^e \)

\[
q_{i,t}^{t,e}(c) = \begin{cases} 
1 & c_i < c_j \forall j \neq i \\
0 & \sim 
\end{cases}
\]  

(1)

- Social cost:

\[
C(\Gamma^e, I) = n \int_C c[1 - F(c)]^{n-1} f(c)dc \\
+ \int_C c[1 - F(c)]^{n-1} g(c, I)dc \\
+ (n - 1) \int_C c[1 - F(c)]^{n-2}[1 - G(c, I)]f(c)dc \\
+ \Psi(I)
\]

(2)
Socially Efficient Investment

The planner solves \( \min_{I \geq 0} C(\Gamma^e, I) \)

Proposition

The socially efficient level of investment, \( I^e \), is the solution to

\[
\max_{I \geq 0} \int_C [1 - F(c)]^{n-1} G(c, I) dc - \Psi(I) \tag{3}
\]

Furthermore, it can be induced using two SPA regardless of the observability of the investment decision.

- Observe that \( (3) \Leftrightarrow \max_{I \geq 0} \int_C [1 - F(c)]^{n-1} \frac{G(c, I)}{g(c, I)} g(c, I) dc - \Psi(I) \)
- Hidden investment: \( I^e \in \arg \max_{I \geq 0} \int_C \Pi^2_e(c, c) g(c, I) dc - \Psi(I) \) and

\[
\Pi^2_e(c, c) = \Pi^2_e(\bar{c}, \bar{c}) + \int_c^\bar{c} Q^2_e(s) ds
\]
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Cost Minimization Under Full Commitment

- Buyer must purchase two goods sequentially at the lowest possible cost.
- She can commit to the rules of both auctions before these take place.
- Suppose that investment is observable.
Notation

\[ T^1_i(c'_i) = \int_{C_{-i}} t^1_i(c'_i, c_{-i}) f^{n-1}_{c_{-i}} dc_{-i} \]

\[ Q^1_i(c'_i) = \int_{C_{-i}} q^1_i(c'_i, c_{-i}) f^{n-1}_{c_{-i}} dc_{-i} \]

\[ \Pi^1_i(c_i, c'_i, I; I) = T^1_i(c'_i) - c_i Q^1_i(c'_i) + Q^1_i(c'_i) \int_C \Pi^2_w(c, c; I) g(c, I) dc \]

\[ + [1 - Q^1_i(c'_i)] \int_C \Pi^2_{\ell,i}(c, c; I) f(c) dc \]

\[ \Pi^2_w(c, c'; I) = T^2_w(c'; I) - c Q^2_w(c'; I) \] (4)
The Buyer’s Problem

The buyer minimizes

\[
C = \sum_{i=1}^{n} \int_{C} T_{i}^{1}(c) f(c) dc \\
+ \int_{C} T_{w}^{2}(c; I) g(c, I) dc + \sum_{j \neq w} \int_{C} T_{\ell,j}^{2}(c; I) f(c) dc
\]  

subject to

- Incentive-compatibility constraints
- Individual rationality (i.e. voluntary participation)
Incentive Compatibility ($I$ is observable)

\[ IC_o : \begin{cases} 
\Pi^2_w(c_w, c'_w; I) \geq \Pi^2_w(c_w, c'_w; I), \forall c_w, c'_w \in C. \\
\Pi^2_{\ell,i}(c_i, c_i; I) \geq \Pi^2_{\ell,i}(c_i, c'_i; I), \forall c_i, c'_i \in C, \forall i \neq w. \\
\Pi^1_i(c_i, c_i, I; I) \geq \Pi^1_i(c_i, c'_i, I; I), \forall c_i, c'_i \in C, \forall i \in N.
\end{cases} \]

Lemma

A mechanism $\Gamma(I)$ is IC if and only if

(i) $Q^1_i(\cdot)$ is non increasing and, for all $c_i \in C$,

\[ \Pi^1_{i,I}(c_i, c_i) = \Pi^1_{i,I}(\bar{c}, \bar{c}) + \int_{c_i}^{\bar{c}} Q^1_i(s; I) ds \]

(ii) $Q^2_k(\cdot; I)$ is non increasing, $k = w, (\ell, i), i \neq w, i \in N$,

\[ \Pi^2_k(c_k, c_k; I) = \Pi^2_k(\bar{c}, \bar{c}; I) + \int_{c_k}^{\bar{c}} Q^2_k(s; I) ds. \]
Participation Constraints

- Participation in the second period is ensured by assuming that

\[ PC^2(I) : \left\{ \begin{array}{l}
\Pi_w^2(c_w, c_w; I) - \Psi(I) \geq 0, \quad \forall c_w \in C \\
\Pi_{\ell, i}^2(c_i, c_i; I) \geq 0, \quad \forall c_i \in C, \ i \neq w.
\end{array} \right. \]

- We follow Pesendorfer and Jofre-Bonet (2014):

\[ PC^1(I) : \Pi_i^1(c_i, c_i, I; I) \geq \int_C \Pi_{\ell, i}^2(c, c; I) f(c) dc, \quad \forall c_i \in C, \ \forall i \in N, \]

Intuition:

- Buyer wants to induce the participation of all sellers in both auctions
- But she cannot prevent the participation at \( t = 2 \) of a seller that skipped the first auction
Optimal Mechanism

Proposition

Suppose that the buyer wants to implement a level $I \geq 0$. The cost-minimizing mechanism, $\Gamma^*(I)$, is given by


ermend\=1\ldots n \quad q_1^*(c_1, \ldots, c_n) = \mathbb{1}\{c_i < c_j, \forall j \neq i\},
ermend\=1\quad w \quad q_2^*(c_w, c_{-w}) = \mathbb{1}\{c_w < c_i + (1 + \frac{1}{n-1}) \frac{F(c_i)}{f(c_i)}, \forall i \neq w\},
ermend\=1\ldots n \quad t_1^*(c_1, \ldots, c_n) = \mathbb{1}\{c_i < k(c_i), \forall i \neq w\} \min \{k(c_i); i \neq w\},
ermend\=1\quad i \quad t_i^1(c_i, c_{-i}; I) = \mathbb{1}\{c_i < c_j, \forall j \neq i\} \left[\min \{c_j; j \neq i\} - (\Pi^2_w(I) - \Psi(I) - \Pi^2_\ell(I))\right] - \Pi^2_\ell(I)
ermend\=1\ldots n \quad t_i^2(c_1, \ldots, c_n) = \mathbb{1}\{c_i < (1 + \frac{1}{n-1}) \frac{F(c_i)}{f(c_i)}, \forall i \neq w\},

where $k(c) := c + \left(1 + \frac{1}{n-1}\right) \frac{F(c)}{f(c)}$ and

$$
\Pi^2_w(I) := \int_C \Pi^2_w(c, c; I) g(c, I) dc \quad \text{and} \quad \Pi^2_\ell(I) := \int_C \Pi^2_\ell(c, c; I) f(c) dc.
$$
Intuition and Remarks

- First auction is efficient; the second is inefficient (advantage gap)
- $\Gamma^*(I)$ is optimal even when $\Psi \equiv 0$. Intuition for the bias then?
  - Incentive to reduce $\Pi^2_\ell(I)$ so as to relax $\Pi^1_i(c_i, c_i, I; I) \geq \Pi^2_\ell(I)$
  - $t^1_i(c_i, c_{-i}; I) = \mathbb{1}_{\{c_i < c_j, \forall j \neq i\}} \left[ \min\{c_j; j \neq i\} - (\Pi^2_w(I) - \Pi^2_\ell(I) - \Psi(I)) \right] - \Pi^2_\ell(I)$
  - Transfer to the winner at $t = 1$ is reduced by $\Pi^2_w(I) - \Pi^2_\ell(I) \to \text{Buyer}$ extracts this extra rent, i.e., increased competition at $t = 1$

- Advantage gap $k(c) = c + \left(1 + \frac{1}{n-1}\right) \frac{F(c)}{f(c)}$:
  - Is independent of $G(\cdot, I)$
  - Never disappears: $k(c) \to c + \frac{F(c)}{f(c)}$ as $n \to \infty$: Isolates the cost-smoothing property of dynamic auctions (In fact, $I^*(n) \to 0$ as $n \to \infty$)
Optimal Investment

Proposition

When investment is observable, the buyer chooses an investment level $I^* > 0$ that solves

$$\max_{I \geq 0} \int_{C} [1 - F(k^{-1}(c))]^{n-1} \frac{G(c, I)}{g(c, I)} g(c, I) dc - \Psi(I),$$

(6)

where $k(c) = c + \left(1 + \frac{1}{n-1}\right) \frac{F(c)}{f(c)}$, $c \in C$. Moreover, $I^* > I^e$, so **over-investment** occurs.

**Intuition:** The winner gets the second project more often that under the efficient mechanism, i.e. $1 - F(k^{-1}(c)) > 1 - F(c)$, which is costly.

Hence, it is optimal to make him win with an even lower average cost.
Hidden Investment: Constraints

- Incentive compatibility:

\[ I \in \arg \max_{K \geq 0} \int C \Pi_2(w, c; I) g(c, K) dc - \Psi(K) \]

\[ \Pi^2_w(c_w, c'_w; I) \geq \Pi^2_w(c'_w, c'_w; I), \ \forall c_w, c'_w \in C \]

\[ \Pi^2_{\ell,i}(c_i, c'_i; I) \geq \Pi^2_{\ell,i}(c'_i, c'_i; I), \ \forall c_i, c'_i \in C, \ \forall i \neq w \]

\[ \Pi^1_i(c_i, c'_i, I; I) \geq \Pi^1_i(c'_i, c'_i, I; I), \ \forall c_i, c'_i \in C, \ \forall i \in N. \]

- Participation constraints: As before
Optimal Contract

Proposition

Γ∗(I∗) induces the winner to invest I∗. Hence, it is optimal when investment is hidden, and I∗ can be implemented at no additional cost. Over-investment occurs.

Proof:

\[
\max_{I \geq 0} \int_{C} \Pi^2_w(c, c; I^*) g(c, I) dc - \Psi(I) = \max_{I \geq 0} \int_{C} Q^2_w(c) g(c, I) dc - \Psi(I)
\]

\[
= \int_{C} [1 - F(k^{-1}(c))]^{n-1} G(c, I) dc - \Psi(I).
\]

Intuition: Incentives on the margin are stepper
Remarks: Full-Commitment Case

- Cost minimization: Investment incentives are aligned under the optimal mechanism.
- Surplus maximization: Investment incentives are aligned under the efficient mechanism.
- Is it the same under any arbitrary mechanism (i.e., a consequence of risk neutrality)? No:

Proposition

Let $n = 2$ and consider the IC mechanism $q_{w,I}^2(c_w, c_l) = 1_{c_w < g(c_l)}$, with $g'(\cdot) \geq 0$, $g(c) = c$ and $g(c) \leq c + 2 \frac{F(c)}{f(c)}$, $\forall c \in C$, with strict inequality on a subset of $C$ with non-zero measure. Then, the buyer chooses an investment level that is larger than the one chosen by the first-period winner.
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Conclusions

- In dynamic contexts, mechanisms serve a dual role:
  - Inter-temporal cost smoothing
  - Induce incentives to invest
- Commitment generates **over-investment** via awarding **advantages** to previous winners
- When the buyer has full commitment not observing investment is irrelevant under optimal contracts (e.g.: cost minimization or surplus maximization). This is not the case when the buyer lacks commitment (**hold-up** effect)
- World is more complicated: although providing an advantage increases investment, it can creates barriers to entry
- **Challenging question:** fully dynamic environment with experience accumulation and history-dependent advantages
Thank you!
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Lack of Commitment

In this case the buyer can change the rules of the second auction after the first one has taken place.

We solve the problem using sequential rationality:

- Observable investment: Stackelberg game in which the buyer treats investment as sunk
- Hidden investment: Simultaneous-move game in which the buyer takes into account the winner’s incentives to invest

Assume $c \mapsto c + \frac{G(c, I)}{g(c, I)}$ is increasing
Observable Investment

- After investment becomes sunk → standard one-shot auction problem (Myerson, 1981) at $t = 2$. Call this mechanism $\hat{\Gamma}^2(I)$.

Proposition

Suppose that winner invests $I \geq 0$. Then, $\hat{\Gamma}^2(I)$ satisfies

$$\hat{q}_w^2(c_w, c_{-w}; I) = \begin{cases} 
1 & c_w + \frac{G(c_w, I)}{g(c_w, I)} < \min_{i \neq w} \left\{ c_i + \frac{F(c_i)}{f(c_i)} \right\} \\
0 & \sim
\end{cases}$$

The investment induced in this setting, $\hat{I}$, satisfies

$$\max_{I \geq 0} V(I) = \int_C \left[ 1 - F(v^{-1}(h(c, I))) \right]^{n-1} G(c, I) dc - \Psi(I)$$

with $h(c, I) = c + \frac{G(c, I)}{g(c, I)}$ and $J(c) = c + \frac{F(c)}{f(c)}$. Hence, $\hat{\Gamma}^2(\hat{I})$ arises in equilibrium, and the winner suffers a disadvantage.
Hidden Investment: Simultaneous-Move Game

- Winner’s action space: \( I \in [0, +\infty) \).
- Buyer’s action space: \( BR_b = \{ \hat{\Gamma}^2(I) \mid I \geq 0 \} \) (rationalizability argument)
- Focus on pure-strategy equilibria

Proposition

*In this context, a pure-strategy equilibrium corresponds to a tuple* \((\hat{\Gamma}^2(\hat{I}), \hat{I}) \in BR_b \times [0, +\infty)\) *that solves*

\[
\begin{align*}
\min_{\hat{\Gamma}(I) \in BR_b} & \quad C^2(\hat{\Gamma}(I), J) \\
\text{s.t.} & \quad J \in \arg\max_{K \geq 0} \int_{C} \hat{Q}_{w, I}(c)G(c, K)dc - \Psi(K)
\end{align*}
\]
Equilibrium Characterization and the Impact of Commitment on Investment Incentives

Proposition

The exists a unique equilibrium is pure-strategies $(\hat{\Gamma}^2(\hat{I}), \hat{I})$ where $\hat{I}$ is characterized by

$$\frac{\partial}{\partial I} \left( \int_C [1 - F(v^{-1}(h(\hat{I}, c)))]^{n-1} G(c, I) dc - \Psi(I) \right) \bigg|_{I=\hat{I}} = 0$$

Proposition

The following ranking holds: $\hat{I} < \hat{I} < I^e < I^*$

Conclusions