Sequential Procurement Auctions and Their Effect on Investment Decisions

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Procurement Auctions

- Markets designed for the purchase of goods (typically of high cost)
- Used both in public and private sector
- Finding ways to **reduce total expenditures** is a question of first-order relevance:
  - OECD countries' public procurement expenditures in 2011 accounted for 19% of their GDP
  - Chile: Transactions performed through *Chilecompra* 10.000 million USD in 2013 ($\sim$ 4% GDP)
  - Also a relevant question in the private sector

Main Features

- These mechanisms are used repeatedly over time
- Tasks sometimes involve a high degree of expertise (*know-how*) $\Rightarrow$ Group of sellers does not change too much
- Sellers can invest in improving their technologies. Specialized tasks $\Rightarrow$ Relationship-specific investments

This Paper

- Two ways through which total expenditures can be reduced are:

(1) Inter-temporal incentives: design of **dynamic mechanisms** that smooth out costs across time

(2) Incentivizing sellers to **invest in cost-reducing technologies**

- We derive the **optimal contract** (i.e. optimal auction + optimal level of investment) chosen by a buyer in an environment where:
  - She must purchase two goods sequentially over time and can fully commit to a two-period mechanism
  - The winner of the first auction can invest in a cost-reducing technology for the second auction

Main Results

- The optimal mechanism gives an **advantage** to the first-period winner in the second auction
  - Advantage decreases with the number of sellers, but it **never disappears**
- In this dynamic setting, commitment induces **over-investment**
- Investment observability is irrelevant for cost minimization and surplus maximization
- More generally, in dynamic environments **awarding advantages**
  - Can induce more competition among sellers $\Rightarrow$ reduce current costs
  - Can incentivize sellers to invest more in cost-reducing technologies $\Rightarrow$ reduce future costs

Literature

- Myerson (1981): Optimal (one-shot) auction design. Better competitors suffer a **disadvantage** in the optimal mechanism
- Arozamena y Cantillón (2004): Investment stage before a one-shot auction takes place. **Underinvestment** in sealed-bid procurement auctions
- Pesendorfer and Jofre-Bonnet (2014): Sequential auctions with exogenous distributions

Contents

Basics

- A **buyer** (she) must purchase two goods sequentially over time
- There are n risk-neutral sellers that are ex-ante identical
- A Seller's cost to produce each good is his **private information**
- Costs are independent across sellers, and also independent across time
- We are interested in **mechanism design**, i.e., the buyer can commit to a two-period mechanism at time zero
  - Since costs are i.i.d. across time, the revelation principle also holds when the buyer lacks commitment

Distributions of Costs

- In the first period a seller's cost is drawn from a c.d.f. $F(\cdot)$, with density $f(\cdot)$ and support $C = [\underline{c}, \overline{c}]$
- First-period losers maintain $F(\cdot)$ for the second period
- The first-period winner instead can invest in a cost-reducing technology between auctions:
  - Investing $I \geq 0 \Rightarrow$ Cost distribution becomes $G(\cdot, I)$, with density $g(\cdot, I)$ and support C
  - Investing is costly: $\Psi : \mathbb{R}_+ \to \mathbb{R}_+$ differentiable, strictly increasing and strictly convex, with $\Psi(0) = \Psi'(0) = 0$.

Regularity Assumptions Over $F(\cdot)$ and $G(\cdot,\cdot)$

Assumption:

(i) $c + F(c)/f(c)$ is strictly increasing in c.

(ii) $F(c) \leq G(c, 0)$ for all $c \in C$.

(iii) For each $c \in C$, $I \mapsto G(c, I)$ is twice continuously differentiable, strictly increasing (**FOSD**) and concave. Furthermore, $\frac{\partial G}{\partial I}(c, 0) > 0$ for all $c \in C$.

Obs: The following are sufficient for (ii) and (iii):

(a) MLRP: For all $c' < c \in C$ and $0 \leq I' < I \in \mathbb{R}$,

$$\frac{f(c')}{f(c)} \leq \frac{g(c', I')}{g(c, I')} < \frac{g(c', I)}{g(c, I)}.$$

(b) Hazard-rate ordering: For all $c \in C$ and $0 \leq I' < I$

$$\frac{g(c, I)}{G(c, I)} \leq \frac{g(c, I')}{G(c, I')} \leq \frac{f(c)}{F(c)}$$

Timeline

- t=0: The rules of both procurement auctions are set
- t=1: First procurement auction takes place
- t=2: (1) Investment takes place. (2) Second procurement auction takes place

Direct Mechanisms

Definition

A direct mechanism that implements $I \geq 0$, $\Gamma(I)$, corresponds to a tuple $\Gamma(I) = (t^1(\,\cdot\,), q^1(\,\cdot\,), t^2_w(\,\cdot\,; I), q^2_w(\,\cdot\,; I), t^2_l(\,\cdot\,; I), q^2_\ell(\,\cdot\,; I))$ where

$$
\begin{aligned}
t^1 &: C^n \to \mathbb{R}^n \text{ (transfer at t=1)} \\
q^1 &: C^n \to \Delta_n \text{ (allocation rule at t=1)} \\
t^2_w(\,\cdot\,; I) &: C^n \to \mathbb{R} \\
q^2_w(\,\cdot\,; I) &: C^n \to [0,1] \\
t^2_\ell(\,\cdot\,; I) &: C^n \to \mathbb{R}^{n-1} \\
q^2_\ell(\,\cdot\,; I) &: C^n \to [0,1]^{n-1}
\end{aligned}
$$

such that $q^2_w(c\,; I) + \sum_{i \neq w} q^2_{\ell,i}(c\,; I) = 1$ for all $c \in C^n$, and such that the first-period winner finds it optimal to invest $I \geq 0$ between auctions.

Contents

Ex-Post Allocative Efficiency

- Planner observes I and realized costs, and maximizes **total surplus**
- Efficient mechanism Γ^e

$$q_i^{t,e}(c) = \begin{cases} 1 & c_i < c_j \; \forall j \neq i \\ 0 & \sim \end{cases} \tag{1}$$

- Social cost:

$$\begin{aligned} \mathcal{C}(\Gamma^e, I) &= n \int_C c[1-F(c)]^{n-1} f(c) dc \\ &+ \int_C c[1-F(c)]^{n-1} g(c,I) dc \\ &+ (n-1) \int_C c[1-F(c)]^{n-2}[1-G(c,I)] f(c) dc \\ &+ \Psi(I) \end{aligned} \tag{2}$$

Socially Efficient Investment

The planner solves $\min_{I \geq 0} \mathcal{C}(\Gamma^e, I)$

Proposition

The socially efficient level of investment, I^e, is the solution to

$$\max_{I \geq 0} \int_C [1 - F(c)]^{n-1} G(c, I) dc - \Psi(I) \qquad (3)$$

Furthermore, it can be induced using two SPA ***regardless of the observability of the investment decision****.*

- Observe that (3) $\Leftrightarrow \max \int_C [1 - F(c)]^{n-1} \frac{G(c,I)}{g(c,I)} g(c, I) dc - \Psi(I)$
- Hidden investment: $I^e \in \arg\max_{I \geq 0} \int_C \Pi_w^{2,e}(c, c) g(c, I) dc - \Psi(I)$ and $\Pi_w^{2,e}(c, c) = \Pi_w^{2,e}(\overline{c}, \overline{c}) + \int_c^{\overline{c}} Q_w^{2,e}(s) ds$

Contents

Cost Minimization Under Full Commitment

- Buyer must purchase two goods sequentially at the lowest possible cost
- She can commit to the rules of both auctions before these take place
- Suppose that investment is observable

Notation

$$
\begin{aligned}
T_i^1(c_i') &= \int\limits_{C_{-i}} t_i^1(c_i', c_{-i}) f^{n-1}(c_{-i}) dc_{-i} \\
Q_i^1(c_i') &= \int\limits_{C_{-i}} q_i^1(c_i', c_{-i}) f^{n-1}(c_{-i}) dc_{-i} \\
\Pi_i^1(c_i, c_i', I; I) &= T_i^1(c_i') - c_i Q_i^1(c_i') + Q_i^1(c_i') \int\limits_{C} \Pi_w^2(c, c; I) g(c, I) dc \\
&\quad + [1 - Q_i^1(c_i')] \int\limits_{C} \Pi_{\ell,i}^2(c, c; I) f(c) dc \\
\Pi_w^2(c, c'; I) &= T_w^2(c'; I) - c Q_w^2(c'; I) \qquad (4)
\end{aligned}
$$

The Buyer's Problem

The buyer minimizes

$$\begin{aligned}\mathcal{C} &= \sum_{i=1}^{n} \int_{C} T_i^1(c) f(c) dc \\ &\quad + \int_{C} T_w^2(c; I) g(c, I) dc + \sum_{j \neq w} \int_{C} T_{\ell,j}^2(c; I) f(c) dc \qquad (5)\end{aligned}$$

subject to

- Incentive-compatibility constraints
- Individual rationality (i.e. voluntary participation)

Incentive Compatibility (I is observable)

$$IC_o : \begin{cases} \Pi_w^2(c_w, c_w; I) \geq \Pi_w^2(c_w, c_w'; I), \ \forall\, c_w, c_w' \in C. \\ \Pi_{\ell,i}^2(c_i, c_i; I) \geq \Pi_{\ell,i}^2(c_i, c_i'; I), \ \forall\, c_i, c_i' \in C, \ \forall\, i \neq w. \\ \Pi_i^1(c_i, c_i, I; I) \geq \Pi_i^1(c_i, c_i', I; I), \forall c_i, c_i' \in C, \forall i \in N. \end{cases}$$

Lemma

A mechanism $\Gamma(I)$ is IC if and only if

(i) $Q_i^1(\cdot)$ *is non increasing and, for all* $c_i \in C$,

$$\Pi_{i,I}^1(c_i, c_i) = \Pi_{i,I}^1(\bar{c}, \bar{c}) + \int_{c_i}^{\bar{c}} Q_i^1(s)ds$$

(ii) $Q_k^2(\,\cdot\,; I)$ *is non increasing,* $k = w, (\ell, i)$, $i \neq w$, $i \in N$,

$$\Pi_k^2(c_k, c_k; I) = \Pi_k^2(\bar{c}, \bar{c}; I) + \int_{c_k}^{\bar{c}} Q_k^2(s; I)ds.$$

Participation Constraints

- Participation in the second period is ensured by assuming that

$$PC^2(I): \begin{cases} \Pi_w^2(c_w, c_w; I) - \Psi(I) \geq 0, \ \forall c_w \in C \\ \Pi_{\ell,i}^2(c_i, c_i; I) \geq 0, \ \forall c_i \in C, \ i \neq w. \end{cases}$$

- We follow Pesendorfer and Jofre-Bonet (2014):

$$PC^1(I): \ \Pi_i^1(c_i, c_i, I; I) \geq \int_C \Pi_{\ell,i}^2(c, c; I) f(c) dc, \ \forall \ c_i \in C, \ \forall \ i \in N,$$

Intuition:

  - Buyer wants to induce the participation of all sellers in both auctions
  - But she cannot prevent the participation at $t = 2$ of a seller that skipped the first auction

Optimal Mechanism

Proposition

Suppose that the buyer wants to implement a level $I \geq 0$. The cost-minimizing mechanism, $\Gamma^*(I)$, is given by

$$
\begin{aligned}
q_i^{1*}(c_1, ..., c_n) &= \mathbb{1}_{\{c_i < c_j,\ \forall j \neq i\}}, \\
q_w^{2*}(c_w, c_{-w}) &= \mathbb{1}_{\left\{c_w < c_i + \left(1 + \frac{1}{n-1}\right)\frac{F(c_i)}{f(c_i)},\ \forall i \neq w\right\}}, \\
t_w^{2*}(c_1, ..., c_n) &= \mathbb{1}_{\{c_w < k(c_i),\ \forall i \neq w\}} \min\{k(c_i); i \neq w\}, \\
t_i^{1*}(c_i, c_{-i}; I) &= \mathbb{1}_{\{c_i < c_j,\ \forall j \neq i\}} \left[\min\{c_j; j \neq i\} - (\Pi_w^{2*}(I) - \Psi(I) - \Pi_\ell^{2*}(I))\right] \\
&\quad -\Pi_\ell^{2*}(I)
\end{aligned}
$$

where $k(c) := c + \left(1 + \frac{1}{n-1}\right)\frac{F(c)}{f(c)}$ *and*

$$\Pi_w^{2*}(I) := \int_C \Pi_w^{2*}(c, c; I) g(c, I) dc \ \ \textit{and} \ \ \Pi_\ell^{2*}(I) := \int_C \Pi_\ell^{2*}(c, c; I) f(c) dc.$$

Intuition and Remarks

- First auction is efficient; the second is inefficient (**advantage gap**)
- $\Gamma^*(I)$ is optimal even when $\Psi \equiv 0$. Intuition for the bias then?
  - Incentive to reduce $\Pi_\ell^2(I)$ so as to **relax** $\Pi_i^1(c_i, c_i, I; I) \geq \Pi_\ell^2(I)$
  - $t_i^{1*}(c_i, c_{-i}; I) =$ $\mathbb{1}_{\{c_i < c_j,\ \forall j \neq i\}} \left[\min\{c_j; j \neq i\} - (\Pi_w^{2*}(I) - \Pi_\ell^{2*}(I) - \Psi(I))\right] - \Pi_\ell^{2*}(I)$
    - Transfer to the winner at $t = 1$ is reduced by $\Pi_w^{2*}(I) - \Pi_\ell^{2*}(I) \rightarrow$ **Buyer extracts this extra rent, i.e., increased competition at** $t = 1$
- Advantage gap $k(c) = c + \left(1 + \frac{1}{n-1}\right) \frac{F(c)}{f(c)}$:
  - Is independent of $G(\cdot, I)$
  - Never disappears: $k(c) \rightarrow c + \frac{F(c)}{f(c)}$ as $n \rightarrow \infty$: **Isolates the cost-smoothing property of dynamic auctions** (In fact, $I^*(n) \rightarrow 0$ as $n \rightarrow \infty$)

Optimal Investment

Proposition

When investment is observable, the buyer chooses an investment level $I^* > 0$ that solves

$$\max_{I \geq 0} \int_C [1 - F(k^{-1}(c))]^{n-1} \frac{G(c, I)}{g(c, I)} g(c, I) dc - \Psi(I), \tag{6}$$

where $k(c) = c + \left(1 + \frac{1}{n-1}\right) \frac{F(c)}{f(c)}$, $c \in C$. *Moreover,* $I^* > I^e$, *so* ***over-investment*** *occurs.*

Intuition: The winner gets the second project more often that under the efficient mechanism, i.e. $1 - F(k^{-1}(c)) > 1 - F(c)$, which is costly. Hence, it is optimal to make him win with an even lower average cost

Hidden Investment: Constraints

- Incentive compatibility:

$$IC_h : \begin{cases} I \in arg\max_{K \geq 0} \int_C \Pi^2_w(c, c; I) g(c, K) dc - \Psi(K) \\ \Pi^2_w(c_w, c_w; I) \geq \Pi^2_w(c_w, c'_w; I), \ \forall\, c_w, c'_w \in C \\ \Pi^2_{\ell,i}(c_i, c_i; I) \geq \Pi^2_{\ell,i}(c_i, c'_i; I), \ \forall\, c_i, c'_i \in C, \ \forall\, i \neq w \\ \Pi^1_i(c_i, c_i, I; I) \geq \Pi^1_i(c_i, c'_i, I; I), \ \forall c_i, c'_i \in C, \ \forall i \in N. \end{cases}$$

- Participation constraints: As before

Optimal Contract

Proposition

$\Gamma^*(I^*)$ induces the winner to invest I^*. Hence, it is optimal when investment is hidden, and I^* can be implemented at no additional cost. Over-investment occurs.

Proof:

$$\max_{I \geq 0} \int\limits_C \Pi_w^{2*}(c, c; I^*) g(c, I) dc - \Psi(I) = \max_{I \geq 0} \int\limits_C Q_w^{2*}(c) g(c, I) dc - \Psi(I)$$

$$= \int\limits_C [1 - F(k^{-1}(c))]^{n-1} G(c, I) dc - \Psi(I).$$

Intuition: Incentives on the margin are stepper

Remarks: Full-Commitment Case

- Cost minimization: Investment incentives are **aligned** under the optimal mechanism
- Surplus maximization: Investment incentives are **aligned** under the efficient mechanism
- Is it the same under any arbitrary mechanism (i.e., a consequence of risk neutrality)? **No**:

Proposition

Let $n = 2$ and consider the IC mechanism $q^2_{w,I}(c_w, c_l) = \mathbb{1}_{c_w < g(c_l)}$, with $g'(\cdot) \geq 0$, $g(\underline{c}) = \underline{c}$ and $g(c) \leq c + 2\frac{F(c)}{f(c)},\ \forall c \in C$, with strict inequality on a subset of C with non-zero measure. Then, the buyer chooses an investment level that is larger than the one chosen by the first-period winner.

Lack of Commitment

Contents

Conclusions

- In dynamic contexts, mechanisms serve a dual role:
  - Inter-temporal cost smoothing
  - Induce incentives to invest
- Commitment generates **over-investment** via awarding **advantages** to previous winners
- When the buyer has full commitment not observing investment is irrelevant under optimal contracts (e.g.: cost minimization or surplus maximization). This is not the case when the buyer lacks commitment (**hold-up** effect)
- World is more complicated: although providing an advantage increases investment, it can creates barriers to entry
- **Challenging question:** fully dynamic environment with experience accumulation and history-dependent advantages

Thank you!

Contents

Lack of Commitment

- In this case the buyer can change the rules of the second auction after the first one has taken place
- We solve the problem using sequential rationality:
  - Observable investment: Stackelberg game in which the buyer treats investment as sunk
  - Hidden investment: Simultaneous-move game in which the buyer takes into account the winner's incentives to invest
- Assume $c \mapsto c + \frac{G(c,I)}{g(c,I)}$ is increasing

Observable Investment

- After investment becomes sunk $\rightarrow$ standard one-shot auction problem (Myerson, 1981) at $t = 2$. Call this mechanism $\hat{\Gamma}^2(I)$.

Proposition

Suppose that winner invests $I \geq 0$. Then, $\hat{\Gamma}^2(I)$ satisfies

$$\hat{q}_w^2(c_w, c_{-w}; I) = \begin{cases} 1 & c_w + \frac{G(c_w, I)}{g(c_w, I)} < \min_{i \neq w} \left\{ c_i + \frac{F(c_i)}{f(c_i)} \right\} \\ 0 & \sim \end{cases}$$

The investment induced in this setting, $\widehat{I}$, satisfies

$$\max_{I \geq 0} V(I) = \int_C [1 - F(v^{-1}(h(c, I))]^{n-1} G(c, I) dc - \Psi(I)$$

with $h(c, I) = c + \frac{G(c,I)}{g(c,I)}$ y $J(c) = c + \frac{F(c)}{f(c)}$. Hence, $\hat{\Gamma}^2(\hat{I})$ arises in equilibrium, and the winner suffers a **disadvantage**

Hidden Investment: Simultaneous-Move Game

- Winner's action space: $I \in [0, +\infty)$.
- Buyer's action space: $BR_b = \{\hat{\Gamma}^2(I) |\ I \geq 0\}$ (rationalizability argument)
- Focus on **pure-strategy equilibria**

Proposition

In this context, a pure-strategy equilibrium corresponds to a tuple $(\hat{\Gamma}^2(\hat{\hat{I}}), \hat{\hat{I}}) \in BR_b \times [0, +\infty)$ *that solves*

$$\begin{cases} \min_{\hat{\Gamma}(I) \in BR_b} \ \mathcal{C}^2(\hat{\Gamma}(I), J) \\ s.t. \quad J \in arg \max_{K \geq 0} \int_C \widehat{Q}^2_{w,I}(c) G(c, K) dc - \Psi(K) \end{cases}$$

Equilibrium Characterization and the Impact of Commitment on Investment Incentives

Proposition

The exists a unique equilibrium is pure-strategies $(\hat{\Gamma}^2(\hat{\hat{I}}), \hat{\hat{I}})$ *where* $\hat{\hat{I}}$ *is characterized by*

$$\frac{\partial}{\partial I}\left(\int_C [1 - F(v^{-1}(h(\hat{\hat{I}}, c)))]^{n-1} G(c, I) dc - \Psi(I)\right)\Bigg|_{I=\hat{\hat{I}}} = 0$$

Proposition

The following ranking holds: $\hat{I} < \hat{\hat{I}} < I^e < I^*$

▸ Conclusions